

# Standard Model Higgs boson mass from inflation: two loop analysis

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**ABSTRACT:** We extend the analysis of [1] of the Standard Model Higgs inflation accounting for two-loop radiative corrections to the effective potential. As was expected, higher loop effects result in some modification of the interval for allowed Higgs masses  $m_{\min} < m_H < m_{\max}$ , which somewhat exceeds the region in which the Standard Model can be considered as a viable effective field theory all the way up to the Planck scale. The dependence of the index  $n_s$  of scalar perturbations on the Higgs mass is computed in two different renormalization procedures, associated with the Einstein (I) and Jordan (II) frames. In the procedure I the predictions of the spectral index of scalar fluctuations and of the tensor-to-scalar ratio practically do not depend on the Higgs mass within the admitted region and are equal to  $n_s = 0.97$  and  $r = 0.0034$  respectively. In the procedure II the index  $n_s$  acquires the visible dependence on the Higgs mass and goes out of the admitted interval at  $m_H$  below  $m_{\min}$ . We compare our findings with the results of [2].

**KEYWORDS:** Inflation, Higgs boson, Standard Model, Variable Planck mass, Non-minimal coupling.

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## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. The basic assumptions</b>	<b>3</b>
<b>3. Action in the inflationary region</b>	<b>5</b>
<b>4. Two-loop effective potential</b>	<b>7</b>
<b>5. Renormalization group equations</b>	<b>8</b>
<b>6. The procedure for computations of inflationary parameters</b>	<b>10</b>
<b>7. Numerical results</b>	<b>12</b>
<b>8. Comparison with refs. [1, 2]</b>	<b>17</b>
<b>9. Conclusions</b>	<b>20</b>
<b>A. Initial conditions</b>	<b>20</b>

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## 1. Introduction

A theory of inflation provides an attractive explanation of the basic properties of the Universe, including its flatness, homogeneity and isotropy (for a recent review see [3]). In addition, it gives a natural source for generation of an almost flat spectrum of fluctuations, leading to the structure formation (for a comprehensive discussion see [4]). Many realisations of inflationary scenario introduce new physics between the electroweak and Planck scales, and postulate the existence of a special scalar field — inflaton. Different models of inflation lead to distinct predictions of the main inflationary parameters, such as the spectral index  $n_s$  of the scalar perturbations and the tensor-to-scalar ratio  $r$ . A number of models have been already excluded or are in tension with the cosmological observations of the cosmic microwave background (CMB) [5].

Within the variety of inflationary models there is one which plays a special role. It does not require introduction of any new physics and identifies the inflaton with the Higgs field of the Standard Model (SM) [6]. The key observation which allows such a relation is associated with a possible non-minimal coupling of the Higgs field  $H$  to the gravitational Ricci scalar  $R$ ,

$$\mathcal{L}_{\text{non-minimal}} = \xi H^\dagger H R . \quad (1.1)$$

For large Higgs backgrounds  $\xi h^2 \gtrsim M_P^2$  (here  $M_P = 2.4 \times 10^{18}$  GeV is the Planck scale and  $h^2 = 2H^\dagger H$ ) the masses of all the SM particles *and* the induced Planck mass  $[M_P^{\text{eff}}]^2 = M_P^2 + \xi h^2$  are proportional to one and the same parameter, leading to independence of physical effects on the magnitude of  $h$ . In other words, the Higgs potential in the large-field region is effectively flat and can result in successful inflation. The fact that the non-minimal coupling of the inflaton to Ricci scalar relaxes the requirement for the smallness of the quartic scalar self-interaction was noted already in [7, 8, 9, 10, 11, 12, 13].

This model of inflation is very conservative in a sense that it does not require the existence of new particles and interactions: the term (1.1) is needed for the renormalizability of the SM in the curved space-time. Since only *one* new parameter is introduced (the value of the non-minimal coupling  $\xi$ , fixed by the amplitude of scalar perturbations at the COBE scale [6]) the Higgs inflation is a predictive theory, allowing to fix a number of parameters that can be found by the observations of the CMB. Moreover, the model does not have a problem of the graceful exit from the inflationary regime and leads to high values of the reheating temperature,  $T_r \sim 10^{13}$  GeV [14]. The value of  $\xi$ , necessary for successful inflation, is required to be rather large,  $\xi \sim 10^3 - 10^4$ .

Due to the minimal character of the Higgs inflation it can be ruled out by the CMB observations *and* by particle physics experiments, especially those at the LHC. In refs. [15, 16] we conjectured that for the Higgs boson to play a role of the inflaton the SM has to be a consistent field theory all the way up to the Planck scale. This puts the mass of the SM Higgs to a specific interval. The mass is constrained from above by the requirement that no Landau pole appears in the scalar self-coupling of the Higgs boson below the Planck scale (see [17, 18, 19, 20] and references therein). The lower limit comes from the demand that the electroweak vacuum is stable for all scalar field values below  $M_P$  (see [21, 22, 23, 24, 25, 26] and references therein). Our results were challenged in [27], claiming that radiative corrections to the tree Higgs potential are large in the inflationary region and that the spectral index  $n_s$  of scalar fluctuations is only consistent with WMAP observations for values of the Higgs mass  $m_H \gtrsim 230$  GeV. The latter conclusions were shown to be modified drastically by the running of the coupling constants in [1] and [2], containing the one-loop and two-loop renormalization group (RG) improved analysis respectively. A similar remark was also made in [28].

The aim of the present work is to upgrade the computation of [1] to the two-loop level. This allows to check the stability of our conclusions against radiative corrections and to make a proper comparison with [2], where two-loop computation was presented.

The paper is organised as follows. In section 2 we discuss the basic assumptions and elaborate on the possible choices of normalization point for radiative corrections. In section 3 we consider the action of the Standard Model in the inflationary region, in section 4 we compute the two-loop effective potential in the inflationary region. In section 5 we discuss the RG improvement of the effective potential. In section 6 we fix the procedure for computing the radiative corrections to the inflationary potential. In section 7 we present the numerical results. In section 8 we discuss the difference between our analysis and that of ref. [2]. Section 9 is conclusions. We use the same notations as in [1]. The present work can be considered as a continuation of this paper. In particular, we omit all the discussions

already made in [1].

## 2. The basic assumptions

It is a challenge to invent a self-consistent framework for computation of radiative corrections to inflation in any type of field theory. The reason is associated with non-renormalizable character of gravity. To clarify this point, let us consider the most popular line of reasoning which is based on the following logic (see, e.g. [29, 30]), related to construction of effective field theories.

Take some field theory coupled to gravity and consider cross-sections of different reactions (such as scalar-scalar or graviton-graviton scattering) computed in the lowest order of perturbation theory. Find the lowest energy  $\Lambda$  at which one of these cross-sections hits the unitarity bound. Call  $\Lambda$  the “ultraviolet cutoff”. The fact that perturbation theory breaks down for energies above  $\Lambda$  may then signal that the theory under consideration is not a fundamental theory, but an effective one, valid only at momenta smaller than the ultraviolet cutoff  $\Lambda$ .<sup>1</sup> If true, the initial Lagrangian *must* be modified by adding to it all sorts of higher-dimensional operators, suppressed by the scale  $\Lambda$ . Now, if the addition of these higher dimensional operators with coefficients of the order of one spoils the effect under consideration, it is said that the realisation of a particular phenomenon (inflation in our case) is “unnatural”. At the same time, if one finds some symmetry principle which keeps the unwanted contributions of higher dimensional operators small or zero, then it is said that the corresponding effect is “natural”.<sup>2</sup>

For the values of the non-minimal couplings  $\xi \gg 1$  the “cutoff” scale, determined by the method described above, is of the order of:<sup>3</sup>

$$\Lambda_\xi \sim \frac{M_P}{\xi} \ll M_P . \quad (2.1)$$

If the higher order operators, suppressed by  $\Lambda_\xi$ , are added to the SM with coefficients of the order of one, they spoil the Higgs-inflation, and, according to the definition discussed above, make it “unnatural” [29, 30]. In other words, the inflationary predictions depend in a sensitive way on the coefficients with which these operators appear.

The point of view we take in this paper is very much different from the reasoning described above. Contrary to [29, 30], we will *assume* that the breaking of perturbation theory, constructed near the SM vacuum at energies greater than  $\Lambda$ , is not a signal of existence of new physics which should replace the SM, but rather an indication of a new physical phenomenon — strong coupling, which should be treated *inside* the SM by non-perturbative methods (such as resummation of different diagrams, lattice simulations, etc.).

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<sup>1</sup>This type of arguments were very successful for construction of renormalizable gauge theories with spontaneous symmetry breaking [31] in general and the SM in particular, starting from four-fermionic Fermi interaction.

<sup>2</sup>Of course, the real physics question is not whether this or that theory is “natural” but whether it is realised in Nature. The small value of the scale of the electroweak symmetry breaking in comparison with the Planck scale is considered to be extremely “unnatural”. The same is true for the cosmological constant. Nevertheless, these are the values we have to live with.

<sup>3</sup>This estimate was done by S. Sibiryakov (private communication) and later in [29, 30].

Put it in other words, we will assume that the SM is valid for all momenta smaller than the Planck mass  $M_P$ . Therefore, no higher dimensional operators, suppressed by  $\Lambda_\xi$  will be added<sup>4</sup>.

Fortunately, the presence of the strong coupling at energies higher than  $\Lambda_\xi$  in scattering of particles defined as excitations above the *SM ground state* (with small vacuum expectation value of the Higgs field) does not prevent the reliable computation of the inflationary potential. As we will see (section 3), in the inflationary region, for the Higgs field values  $h_{\text{inf}} \sim M_P/\sqrt{\xi}$ , an adequate description of the system is given by the so-called chiral electroweak theory [32] (SM with the frozen radial Higgs mode). This theory is not renormalizable and has an ultraviolet cutoff  $\Lambda_{\text{inf}} \simeq h_{\text{inf}}$ , defined with the use of the procedure, discussed above.  $\Lambda_{\text{inf}}$  is associated with the scattering of the longitudinal intermediate vector bosons.<sup>5</sup> At the same time, the typical energy saturating the loop diagrams appearing in the computation of the effective potential is of the order of the masses of the SM particles in the Higgs background (dimensional regularisation is assumed) and thus is smaller than the cutoff by the magnitude of gauge or Yukawa coupling constants. In other words, the contribution of the strongly interacting domain of energies to the low-energy observables such as the effective potential is suppressed.

In spite of the fact that we do not include higher-order operators to the computation, the effective potential *cannot* be fixed unambiguously. In section 3 of [1] we discussed two distinct possibilities. In the first one (referred as prescription I) the normalization point  $\mu$  (coinciding, for example, with t'Hooft-Veltman parameter in dimensional regularisation) of the loop integrals is chosen to be proportional to the coefficient in front of the scalar curvature. In the Jordan frame this gives

$$\mu^2 \propto M_P^2 + \xi h^2, \quad (2.2)$$

and, consistently, in the Einstein frame

$$\mu^2 \propto M_P^2. \quad (2.3)$$

With these choices the computations in Jordan and Einstein frames are equivalent both at classical and quantum levels. The prescription II corresponds to the Jordan frame normalization point given by (2.3). For consistency, one has to choose then the normalization point in the Einstein frame as

$$\mu^2 \propto M_P^4/(M_P^2 + \xi h^2). \quad (2.4)$$

In short, the prescription I is “standard” (field-independent) in the Einstein frame, whereas the prescription II is “standard” in the Jordan frame.

A possible physics behind the choice II is related to an idea that the Jordan frame is the one in which “distances are measured” [27] and that in this frame there exist a fundamental

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<sup>4</sup>As has been shown in [14], the higher order operators suppressed by  $M_P$  are harmless for inflation.

<sup>5</sup>Not surprisingly, the energy at which the tree cross-sections start to exceed the unitarity bound, depends on the background field. For example, for the graviton-graviton scattering the corresponding cutoff is  $[M_P^{\text{eff}}]^2$ , rather than  $M_P^2$ .

cut-off, independent on background scalar field. A possible physics leading to the choice I is related to quantum scale invariance, discussed in [33], though the prescription II can be easily realised in scale-invariant theories as well.<sup>6</sup>

To elucidate the difference between these two prescriptions let us consider the limit of large Higgs fields,  $\xi h^2 \gg M_P^2$ . In this case the dimensionful parameters  $M_P^2$  and the Higgs mass can be neglected, and the Lagrangian of the electroweak theory with gravity incorporated acquires a new symmetry (at the classical level) — the scale invariance,  $\phi(x) \rightarrow \lambda^\alpha \phi(\lambda x)$ , where  $\phi$  is a generic notation for any field with canonical mass dimension  $\text{GeV}^\alpha$ . Note that in the presence of gravity the scale invariance does not ensure conformal invariance, since  $\xi \neq -\frac{1}{6}$ . The fate of the dilatation symmetry at the quantum level depends on the way the infinities arising in loop computations are regularized and subtracted.

The use of standard regularization and renormalization schemes (such as dimensional or Pauli-Villars) leads to the trace anomaly in energy-momentum tensor and thus to the breaking of dilatation symmetry (for a review see [34]). The flat direction, existing for the Higgs field in Einstein frame, gets bended owing to radiative corrections. This is the prescription II. If gravity is taken away, it leads to a *renormalizable* field theory.

A non-conventional approach is to use dimensional regularization ( $n = 4 - 2\epsilon$ ) and keep the scale-invariance intact even if  $\epsilon \neq 0$  [35, 33] (see also [36]) by replacing the t'Hooft-Veltman parameter  $\mu$  by a combination of dynamical fields with matching mass dimension. In this case the theory stays scale-invariant at the quantum level and the flat direction remains intact. This is the prescription I. If gravity is taken away, it leads to an *effective* field theory, valid for momenta transfers up to the effective Planck scale  $\sim \sqrt{\xi} h$  [37].

It is the trace (or conformal) anomaly in the total matter energy-momentum tensor which leads to the dependence of the inflationary predictions on the Higgs mass. The one- and two-loop corrections to the effective potential, that we find in this paper in the inflationary region, account exactly for this anomaly, which is different in prescriptions I and II.

It looks impossible to fix the unique prescription without knowing the physics at the Planck scales. In other words, the determination of inflationary cosmological parameters, such as the spectral index  $n_s$  through parameters of the SM, is subject to an intrinsic uncertainty, related to renormalization procedure. We will do computations with both prescriptions. Fortunately, the numerical difference happens to be small in two-loop approximation, as it was so at one-loop [1].

### 3. Action in the inflationary region

After transformation to the Einstein frame the electroweak Lagrangian is essentially non-linear and is given by

$$\mathcal{L}_{\text{chiral}} = \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{g^2} H_1 - \frac{1}{g'^2} H_2 - L_{W/Z} + L_Y - U(\chi), \quad (3.1)$$

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<sup>6</sup>In the language of [33] the prescription I corresponds to the choice of the dimensional regularisation parameter  $\mu^2 \propto \xi_\chi \chi^2 + \xi_h h^2$ , while prescription II corresponds to the choice  $\mu^2 \propto \chi^2$ , where  $\chi$  is the dilaton field.

where

$$H_1 = \frac{1}{2} \text{tr}[W_{\mu\nu}^2] , \quad (3.2)$$

$$H_2 = \frac{1}{4} B_{\mu\nu}^2 , \quad (3.3)$$

$$L_{W/Z} = \frac{v^2}{4} \text{tr}[V_\mu^2] , \quad (3.4)$$

$$L_Y = i\bar{Q}_{L,R} \not{D} Q_{L,R} - (\frac{y_t v}{\sqrt{2}} \bar{Q}_L \tilde{U} Q_R + \cdots + \text{h.c.}) , \quad (3.5)$$

and  $D$  is the standard covariant derivative for the fermions. The field  $\chi$  is related to the module of the Higgs field  $h$  through the equation

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}} , \quad (3.6)$$

with the conformal factor  $\Omega(h)$  given by

$$\Omega^2(h) = 1 + \frac{\xi h^2}{M_P^2} . \quad (3.7)$$

For  $h \ll M_P/\sqrt{\xi}$  the relation between  $\chi$  and  $h$  is

$$\chi \simeq \frac{h}{2} \left[ \sqrt{1 + 6\xi^2 h^2/M_P^2} + \frac{\text{arcsinh}(\sqrt{6}\xi h/M_P)}{\sqrt{6}\xi h/M_P} \right] , \quad (3.8)$$

and for  $h \gg M_P/\xi$  we have

$$\chi \simeq \sqrt{\frac{3}{2}} M_P \log \Omega^2(h) . \quad (3.9)$$

The parameter  $v$  in the inflationary case is

$$v^2(h) = \frac{h^2}{\Omega^2(h)} . \quad (3.10)$$

The dimensionless Nambu-Goldstone bosons  $\pi^a$  are parametrized in the non-linear form as

$$\mathcal{U} = \exp[2i\pi^a T^a] , \quad T^a = \frac{\tau^a}{2} . \quad (3.11)$$

In the Yukawa Lagrangian we only write explicitly the top quark contribution, and define the quark fields as

$$\bar{Q}_L = (\bar{t}_L, \bar{b}_L) , \quad \tilde{U} = -\tau_2 \mathcal{U}^* \tau_2 , \quad \bar{Q}_R = (\bar{t}_R, 0) , \quad (3.12)$$

where  $\tau_a$  are the Pauli matrices.

The gauge fields and the fields strength are given by

$$V_\mu = (\partial_\mu \mathcal{U}) \mathcal{U}^\dagger + iW_\mu - i\mathcal{U} B_\mu^Y \mathcal{U}^\dagger , \quad (3.13)$$

$$W_\mu = W_\mu^a T^a , \quad B_\mu^Y = B_\mu T^3 , \quad (3.14)$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + i[W_\mu, W_\nu] , \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu . \quad (3.15)$$

The scalar potential is

$$U(\chi) = \frac{\lambda h^4(\chi)}{4\Omega^4}, \quad (3.16)$$

where we have neglected the Higgs mass, irrelevant for analysis of inflation. For  $h \gg M_P/\xi$  we have

$$U(\chi) \simeq \frac{\lambda M_P^4}{4\xi^2} \left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2. \quad (3.17)$$

As has been discussed in [1], in the background of the “small” Higgs fields  $h \lesssim \frac{M_P}{\xi}$ , the action in the Einstein frame coincides with that of the canonical Standard Model. Thus, in this region the computation of the effective potential is straightforward (and in fact not necessary, as the inflation takes place at higher values of the scalar field).

In the inflationary region the Higgs field is of the order

$$h \gtrsim \frac{M_P}{\sqrt{\xi}}, \quad (3.18)$$

corresponding to  $\chi \gtrsim M_P$ . For  $\xi \gg 1$  the Higgs field is essentially massless in the region (3.18), and decouples from all the fields of the SM. The transition between these two regimes is governed by the effective couplings of the deviation  $\delta\chi$  of the field  $\chi$  from its background value  $\chi_c$  to the fields of the SM. The canonical values of the Higgs field couplings to other fields are multiplied by the factor  $q$  given by

$$q = \frac{d[h/\Omega]}{d\chi} = \frac{1}{\Omega} \frac{1}{\sqrt{\Omega^2 + 6\xi^2 h^2/M_P^2}}. \quad (3.19)$$

For small fields  $h \ll M_P/\xi$  the factor  $q$  is equal to the SM value  $q = 1$  and for large fields  $M_P/\xi \ll h \lesssim M_P/\sqrt{\xi}$  it leads to the suppression of Higgs interactions by  $q = M_P/(\sqrt{6}\xi h) \ll 1$ . This effect can also be seen in the Jordan frame [27, 2], where it emerges due to the kinetic mixing between the Higgs field and gravitational degrees of freedom. The relation between the suppression factor  $s$  found in [2] and the factor  $q$  is  $s = q^2\Omega^4$ .

In the limit  $h \rightarrow \infty$  the action (3.1) is nothing but the chiral electroweak theory studied in detail in numerous papers (for a review see, e.g. [38]) plus a massless non-interacting field. It will be used for analysis of radiative corrections to the inflationary potential below.

#### 4. Two-loop effective potential

The effective potential for the field  $\chi$  is gauge-invariant, since  $\chi$  is gauge-invariant itself. However, the computation is most convenient in Landau gauge, in which the Goldstone bosons (hidden in the matrix  $\mathcal{U}$ ) and ghosts are massless, and a number of diagrams can be dropped due to the fact that the vector propagators are transverse [39]. The one-loop contribution to the potential is given by

$$U_1 = \frac{6m_W^4}{64\pi^2} \left( \log \frac{m_W^2}{\mu^2} - \frac{5}{6} \right) + \frac{3m_Z^4}{64\pi^2} \left( \log \frac{m_Z^2}{\mu^2} - \frac{5}{6} \right) - \frac{3m_t^4}{16\pi^2} \left( \log \frac{m_t^2}{\mu^2} - \frac{3}{2} \right), \quad (4.1)$$



where we keep now the constant terms omitted in Eq. (9) of [1], as they are needed for a consistent two-loop analysis. As in [1], the part associated with the (light in the inflationary region) field  $\chi$  is neglected. The background field masses of intermediate bosons ( $m_W$  and  $m_Z$ ), and of  $t$ -quark ( $m_t$ ) are given in Eq. (8) of [1].

To find the potential in two loops, one should first expand  $\mathcal{U}$  with respect to Goldstone fields  $\pi^a$ . It is sufficient to keep first order terms in the Yukawa part (3.5) and second order terms in the gauge part (3.1). After this is done, the computation of the two-loop contribution is straightforward but rather involved. Fortunately, all necessary contributions can be extracted from [40]. This paper found the following representation of the two-loop contribution:

$$U_2 = V_S + V_{SF} + V_{SV} + V_{FV} + V_V , \quad (4.2)$$

where  $V_S$  represents scalar loops,  $V_{SF}$ —the diagrams involving scalars and fermions,  $V_{SV}$ —scalars and vectors,  $V_{FV}$ —fermions and vectors, and  $V_V$ —vectors and ghosts. To get the effective potential for our case one has to take away all diagrams involving the Higgs field, since the interactions of it with itself and all matter fields are suppressed by  $\xi$  in the inflationary region. In addition, all Goldstone masses have to be put to zero. As a result, the following substitutions have to be made:

$$V_S \rightarrow 0 , \quad (4.3)$$

$$V_{SF} \rightarrow m_t^2 \left( m_t^2 \hat{I}(m_t, 0, 0) + \frac{1}{2} \hat{J}(m_t, m_t) \right) , \quad (4.4)$$

$$V_{SV} \rightarrow \frac{1}{4} g^2 \left[ \frac{(1 - 2 \sin^2 \theta)^2}{2 \cos^2 \theta} A(0, 0, m_Z) + A(0, 0, m_W) \right] \\ - g^2 \sin^4 \theta m_Z^2 B(m_Z, m_W, 0) - e^2 m_W^2 B(m_W, 0, 0) , \quad (4.5)$$

whereas  $V_{FV}$  and  $V_V$  stay intact. Explicit expressions for different functions appearing here are quite long and we do not reproduce them, see [40] for all definitions.

The two-loop effective potential derived in this way contains an explicit dependence on the normalization point  $\mu$ . It is, however, spurious as it must be compensated by the running of the coupling constants and the field  $\chi$ . If we knew the RG running of the parameters, the best choice of  $\mu$  would be the background mass of a vector boson or of  $t$ -quark, which will minimize the logarithms in the higher-loop corrections. In the next section we will derive the necessary RG equations, valid for large field values in the inflationary region.

## 5. Renormalization group equations

The change of the scalar field from the border-line of applicability of the canonical Standard Model  $h \sim M_P/\xi$  to the inflationary region  $h_{\text{inf}} \sim M_P/\sqrt{\xi}$  is relatively small. This means that the *one-loop* renormalization improvement of the *two-loop* effective potential will do already a very good job. The main objects which appear in the one-loop effective potential (4.1) are the background masses of  $W$ ,  $Z$  bosons and of  $t$ -quark. Therefore, we need to find the logarithmic running of these masses. It is sufficient to consider their renormalization in

the asymptotic region of large  $h$ , where the tree potential for the scalar field is flat. Here the action (3.1) is just the action of the chiral SM with  $v = M_P/\sqrt{\xi}$ .

To find the running of  $m_Z$ ,  $m_W$  and  $m_t$  it is sufficient to consider the  $WW$ ,  $ZZ$  and  $tt$  two-point functions in the chiral SM. The explicit computation gives (in the  $\overline{\text{MS}}$  subtraction scheme):

$$16\pi^2\mu\frac{\partial}{\partial\mu}m_W^2 = \left(\frac{3g'^2}{2} - \frac{23g^2}{2} - 6y_t^2 + \frac{8g^2n_f}{3}\right)m_W^2, \quad (5.1)$$

$$16\pi^2\mu\frac{\partial}{\partial\mu}m_Z^2 = \left(\frac{40n_fg'^4}{9} + \frac{g'^4}{6} + \frac{8g^2g'^2}{3} - 6y_t^2g'^2 - \frac{23g^4}{2} - 6g^2y_t^2 + \frac{8g^4n_f}{3}\right)\frac{v^2}{4}, \quad (5.2)$$

$$16\pi^2\mu\frac{\partial}{\partial\mu}m_t^2 = -\left(\frac{4g'^2}{3} + 16g_3^2\right)m_t^2, \quad (5.3)$$

where  $n_f = 3$  is the number of fermionic generations.

In fact, it is more convenient to rewrite these equations by introducing the running of the coupling constants  $g'$ ,  $g$ ,  $g_3$ ,  $y_t$ ,  $\lambda$ , and  $\xi$ . As the chiral SM is well studied in the literature, we can extract the necessary results from the existing computations. Reference [41] gives the following equations for the running of  $g'$ ,  $g$ :

$$16\pi^2\mu\frac{\partial}{\partial\mu}g' = \left(\frac{1}{6} - \frac{1}{12} + \frac{20n_f}{9}\right)g'^3, \quad (5.4)$$

$$16\pi^2\mu\frac{\partial}{\partial\mu}g = -\left(\frac{43}{6} + \frac{1}{12} - \frac{4n_f}{3}\right)g^3. \quad (5.5)$$

The number  $\frac{1}{12}$  accounts for the fact that the Higgs field (but not Nambu-Goldstone bosons) decouples from the fields of the SM.

Reference [41] also gives an equation for running of the parameter  $v^2$ , without accounting for the Yukawa couplings. It is not difficult to add the corresponding contribution, what leads to:

$$16\pi^2\mu\frac{\partial}{\partial\mu}v^2 = \left(\frac{3}{2}g'^2 + 3g^2 - 6y_t^2\right)v^2. \quad (5.6)$$

Since  $v^2 \propto \frac{1}{\xi}$  in our case, this can be converted to equation for  $\xi$ :

$$16\pi^2\mu\frac{\partial}{\partial\mu}\xi = -\left(\frac{3}{2}g'^2 + 3g^2 - 6y_t^2\right)\xi. \quad (5.7)$$

Clearly, the running of the strong coupling does not change in the chiral phase, so that

$$16\pi^2\mu\frac{\partial}{\partial\mu}g_3 = -7g_3^2. \quad (5.8)$$

Now, combination of Eq. (5.3) with (5.6) gives an equation for the top Yukawa coupling

$$16\pi^2\mu\frac{\partial}{\partial\mu}y_t = \left(-\frac{17}{12}g'^2 - \frac{3}{2}g^2 - 8g_3^2 + 3y_t^2\right)y_t. \quad (5.9)$$

The equation for  $\lambda$  follows from equations for  $v^2$  and from the fact that the one-loop effective potential  $U(h, \lambda(\mu)) + U_1(h, \mu)$  in the asymptotic region of the Higgs fields must

be independent on  $\mu$ :

$$16\pi^2\mu\frac{\partial}{\partial\mu}\lambda = -6y_t^4 + \frac{3}{8}(2g^2 + (g'^2 + g^2)^2) + (-3g'^2 - 6g^2 + 12y_t^2)\lambda. \quad (5.10)$$

This equation is quite peculiar as it does not contain the term  $\lambda^2$  and thus does not have the Landau-pole behaviour at one loop. It is instructive to provide also the equation for the combination  $\lambda/\xi^2$ , which is the only combination of constants present in the tree level potential (3.17)

$$16\pi^2\mu\frac{\partial}{\partial\mu}\left(\frac{\lambda}{\xi^2}\right) = \frac{1}{\xi^2}\left(-6y_t^4 + \frac{3}{8}(2g^2 + (g'^2 + g^2)^2)\right). \quad (5.11)$$

This equation does not contain terms proportional to  $\lambda$  at all, making the running less important for heavier Higgs masses. It is interesting to note that the right-hand-side of Eq. (5.11) is exactly the same for  $h \lesssim M_P/\xi$ , what can be derived from eqns. (10-14) of [1].

However, this system of RG equations is not complete. The reason is that the chiral SM is not renormalizable, so that in order to remove the divergences one has to add a certain set of counter-terms with the structures different from those already present in (3.1). These operators were found and discussed in detail in a number of papers (see, e.g. a recent work [41]). Only few of these operators are needed for our purposes, namely those which contribute to the renormalization of the mass of  $Z$  boson.<sup>7</sup> These are:<sup>8</sup>

$$\frac{1}{4}\alpha_0 v^2 (\text{tr}[\Upsilon V_\mu])^2 + \frac{1}{2}\alpha_1 B_{\mu\nu} \text{tr}[\Upsilon W^{\mu\nu}],$$

where  $\Upsilon = 2\mathcal{U}T^3\mathcal{U}^\dagger$ . The coefficients  $\alpha_0$  and  $\alpha_1$  obey the RG equations [41]

$$16\pi^2\mu\frac{\partial}{\partial\mu}\alpha_0 = \frac{3}{4}g'^2, \quad 16\pi^2\mu\frac{\partial}{\partial\mu}\alpha_1 = \frac{1}{6} \quad (5.12)$$

and the mass of the  $Z$ -boson is

$$m_Z^2 = \frac{1}{4}((g'^2 + g^2)(1 - 2\alpha_0) - 2g'^2 g^2 \alpha_1) v^2. \quad (5.13)$$

This expression, supplemented by the RG equations (5.4), (5.5), (5.8), (5.7), (5.9), (5.10), (5.12), allows for a one-loop improvement of the 2-loop effective potential in the inflationary region. The initial conditions for the running are to be fixed at the borderline between the SM phase and the SM chiral phase, corresponding to  $\mu \simeq M_P/\xi$ .

## 6. The procedure for computations of inflationary parameters

To determine the inflationary parameters, we generalize the procedure described in [1] to two loop level. It consists of several steps described below.

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<sup>7</sup>Note that the renormalization of  $W$  and  $t$  masses does not require any new counter-terms, at least at the order in the coupling constants we are working with.

<sup>8</sup>A potentially important operator corresponding to  $\alpha_8$  (see [41] for definition) does not contribute at the order in the coupling constants we are interested in.

1. To find the initial conditions for the RG running of the coupling constants at  $\mu = m_t$  we relate the values of the  $\overline{\text{MS}}$ -scheme  $g$ ,  $\lambda$  and  $y_t$  to the pole Higgs and top masses with the use of the formulas given in section 5 of ref. [42] and in the appendix of ref. [43]. The procedures described in these works give almost identical results (see appendix).
2. We replace the system of one-loop RG equations (10-14) of [1] for all three gauge couplings, scalar self-coupling and top-quark Yukawa couplings by the corresponding two-loop equations taken from [44] and reproduced in convenient form in [43]. Solving these equations up to the scale  $M_P/\xi$  gives us the values of parameters at the borderline between the SM and chiral SM.
3. The values of the coupling constants found in the previous step are used as initial values for RG group running in the chiral phase of the EW theory. Since the parameters  $\alpha_0$  and  $\alpha_1$  are absent in the SM, the initial conditions for them are fixed to be zero.<sup>9</sup>
4. The two-loop effective potential is computed in the inflationary region as  $U + U_1 + U_2$ , with all coupling constants taken at scale  $\mu$ , and the  $Z$  boson mass given by (5.13). To optimize the amplitude of higher order corrections, the parameter  $\mu$  is chosen as in [1].

For the renormalization prescription I the value of  $\mu$  is

$$\mu^2 = \kappa^2 m_t^2(\chi) = \kappa^2 \frac{y_t(\mu)^2}{2} \frac{M_P^2}{\xi(\mu)} \left( 1 - e^{-\frac{2\chi}{\sqrt{6}M_P}} \right). \quad (6.1)$$

Here  $\kappa$  is some constant of order one.<sup>10</sup>

For the prescription II we take another value for  $\mu$ :

$$\mu^2 = m_t^2(\chi) \Omega(\chi)^2 = \frac{y_t(\mu)^2}{2} \frac{M_P^2}{\xi(\mu)} \left( e^{\frac{2\chi}{\sqrt{6}M_P}} - 1 \right). \quad (6.2)$$

5. As the potential is fixed, the value of  $\xi$  is chosen in such a way that the correct WMAP5 normalization is reproduced. It is important to stress here that the wave-function normalization for the Higgs field in inflationary domain can be neglected, since the Higgs interactions with matter are suppressed by large value of non-minimal coupling  $\xi$ .

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<sup>9</sup>The interpolation between the SM running of the coupling constants and the running in the chiral SM can be made smooth with the use of  $q$ -factor, defined in (3.19), in analogy with ref. [2]. This leads, however, to negligible modifications of numerics, since  $q(\chi)$  defined in Eq. (3.19) is a rapidly changing function in the region  $h \sim M_P/\xi$ . A simple demonstration of this fact can be made by switching from the SM RG running to the chiral SM one at  $M_P/\sqrt{\xi}$  instead of  $M_P/\xi$ .

<sup>10</sup>For technical reasons instead of expressions (6.1), (6.2) we used for numerics the same formulas, but with the constants  $y_t$ ,  $\xi$  taken at the point  $M_P/\xi$ .

6. The spectral index  $n_s$  and tensor-to scalar ratio  $r$  are computed. To be more specific, for the given potential the field  $\chi$  corresponding to the end of inflation is found, then the field corresponding to 59 e-foldings of inflation is found, then at this scale  $n_s$  and  $r$  are calculated using standard slow-roll formulas.

The downgrade of this procedure to tree mapping at  $\mu = m_Z$ , one-loop running from this point to  $M_P/\xi$ , and the use of one-loop effective potential in the inflationary region *without* RG improvement coincides with the computation carried out in our earlier work [1] up to higher-order terms in coupling constants, due to Eq. (18) of [1].<sup>11</sup> The computation of [27] corresponds to the same procedure with the prescription II but without RG running from  $m_Z$  to  $M_P/\xi$ . In [27] the values of the coupling constants at  $\mu = M_P/\xi$  were taken to be the same as at  $\mu = m_Z$ . The computation [2] corresponds nominally (for a more detailed comparison see section 8) to one-loop mapping at  $\mu = m_t$  and the use of tree effective potential improved with the use of two-loop RG equations up to the inflation scale.

## 7. Numerical results

The results of the computation of the spectral index and tensor-to-scalar ratio using the procedure from the previous section with both prescriptions I and II are shown in Figs. 1, 2. The running of the spectral index  $dn_s/d\ln k$  is always very small, of the order of  $10^{-4}$ .

Let us first comment on the results in the prescription I. The spectral index  $n_s$  is nearly constant, exactly the same as for one-loop computation [1]. The only difference is the shift of the minimal allowed Higgs boson masses to:

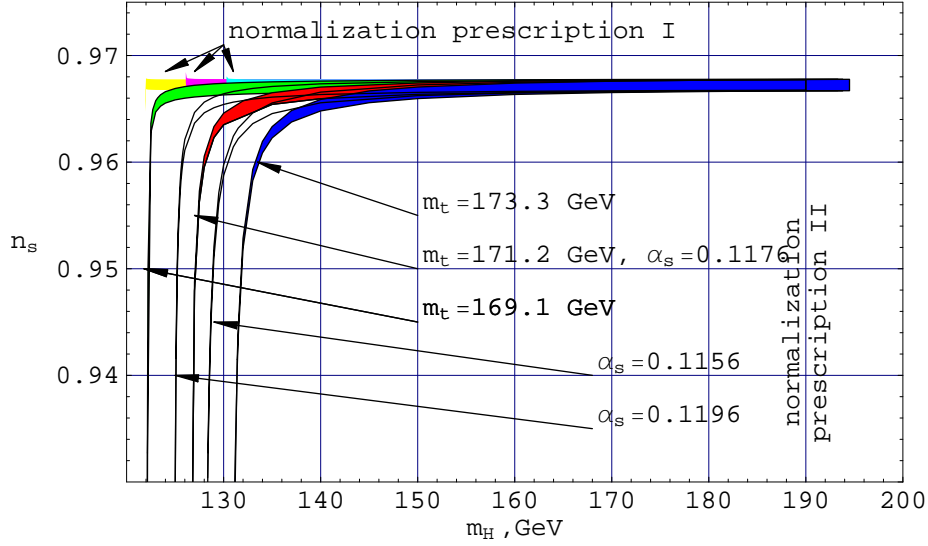
$$m_{\min}^I = \left[ 126.1 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5 \right] \text{ GeV} , \quad (7.1)$$

which happens mainly due to the two-loop running up to the  $M_P/\xi$  scale. One can see small wiggles at the lowest  $m_H$  on the lines (they were more pronounced in the one-loop result). This corresponds to the situation when  $\lambda$  is approaching zero in the inflationary region and the tree level part of the potential becomes comparable with the loop corrections. It is unclear whether this feature is a real physical effect or a result of approximation.

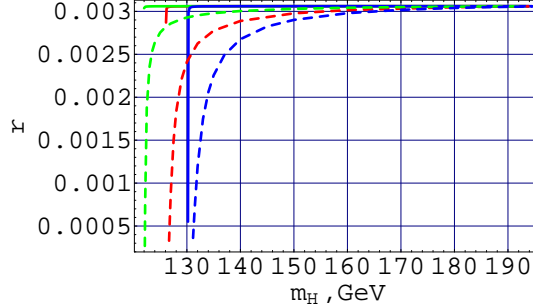
At the high  $m_H$ , contrary to the one-loop analysis, the solution to two-loop RG equations for the scalar self-coupling does not show up a Landau-pole behaviour. Though  $\lambda$  enters into strong-coupling region, it stays finite. Of course, perturbation theory cannot be trusted in this case. We believe that inflation is hardly possible in this situation. The lattice simulations of pure scalar theory [45] show that the corresponding theory has nothing in common with a continuum strongly coupled field theory. Requiring that one should

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<sup>11</sup>The RG equations in the chiral phase of the EW theory found in the present work are different from the RG equations used in [1] in this domain. However, the one-loop running of the combination of couplings  $\lambda/\xi^2$ , most relevant for determination of the inflationary potential, is exactly the same in both phases, see Eq. (5.11).



**Figure 1:** Spectral index  $n_s$  depending on the Higgs mass  $m_H$ , calculated with the RG enhanced effective potential. Nearly horizontal coloured stripes correspond to the normalization prescription I and different  $m_t$ . Green, red, and blue stripes give the result with normalization prescription II for different  $m_t$  and  $\alpha_s = 0.1176$ , two white regions correspond to different  $\alpha_s$  and  $m_t = 171.2$  GeV. The width of the stripes corresponds to changing the number of e-foldings between 58 and 60, or approximately one order of magnitude in reheating temperature.

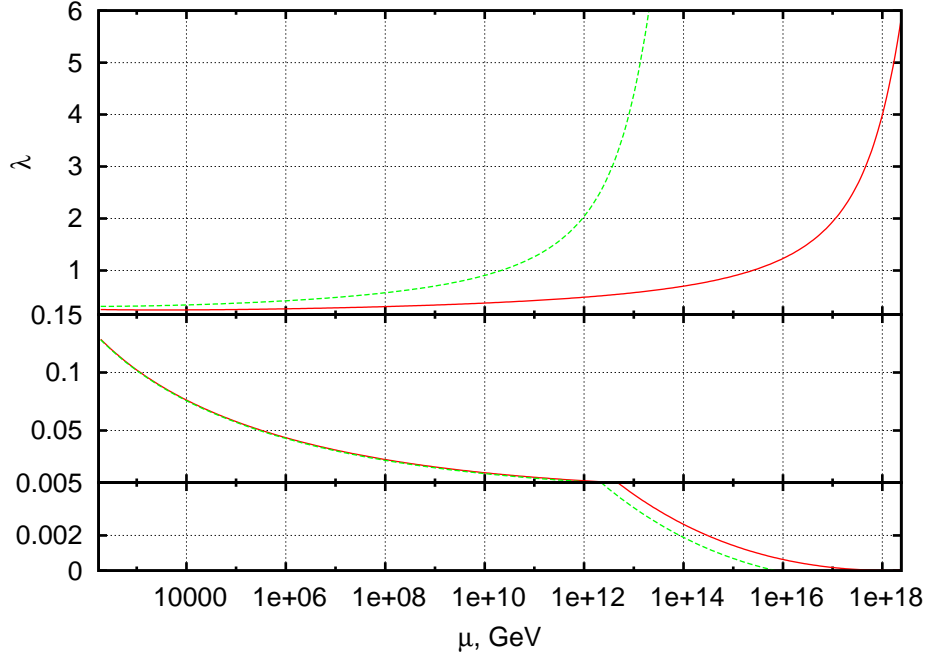


**Figure 2:** Tensor-to-scalar ratio  $r$  depending on the Higgs mass  $m_H$ , calculated with the RG enhanced effective potential. Nearly horizontal solid lines correspond to the normalization prescription I. Green, red, and blue dashed lines give the result with normalization prescription II for  $m_t = 169.1, 171.2, 173.3$  GeV. Dependence on the number of e-foldings is very small.

have the weak coupling (somewhat arbitrary we set the requirement  $\lambda < 6$ ) in the region  $M_P/\xi$  we obtain, in 2-loop approximation,

$$m_{\max}^I = \left[ 193.9 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.1176}{0.002} \times 0.1 \right] \text{ GeV} . \quad (7.2)$$

Note that the requirement to have weak coupling up to  $\mu = M_P$  is somewhat stronger,  $m_H < 173.5$  GeV (for central values of  $m_t$  and  $\alpha_s$ ). Also, the requirement that  $\lambda(\mu) > 0$  for  $\mu < M_P$  is just slightly stronger, than (7.1), giving the interval of the Higgs masses for



**Figure 3:** The running of the Higgs coupling constant  $\lambda$  up to the high scale in the canonical SM. Solid line graphs correspond to  $m_H = 174$  GeV and  $m_H = 126.3$  GeV, leading to  $\lambda = 6$  at  $M_P$ , or  $\lambda$  touching 0 at a scale slightly below (these values represent the window in which the SM is valid all the way to the Planck scale, for central values of  $m_t$  and  $\alpha_s$ ). Dashed graphs correspond to  $m_H = 194$  GeV and  $m_H = 126.1$  GeV, leading to strong coupling or zero at  $M_P/\xi$  (for  $\xi = 11600$  and 704, respectively). The lower part of the graph is scaled in the vertical direction for visibility.

which the SM remains a good quantum field theory all the way up to the Planck scale, see Fig. 3.

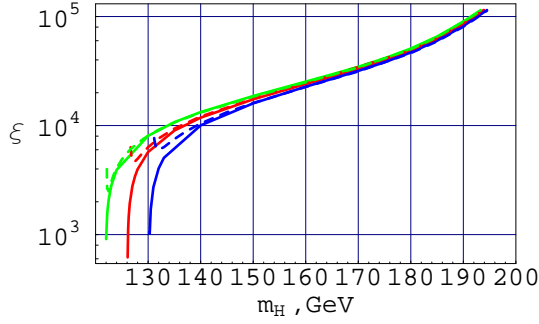
Figures 4 and 5 give for reference the values of  $\xi$  and  $\lambda$  at the scale  $M_P/\xi$ . One can see, that the tree level COBE normalization requirement [14] holds very well for high Higgs masses, and becomes violated at lower ones, see Fig. 6.

For prescription II the behaviour is qualitatively similar to the one-loop calculation in [1]. Numerically, we have the overall shift in the Higgs mass region of 10 GeV, corresponding to the difference between one- and two-loop running of the coupling constants up to the inflationary region  $M_P/\xi$ . Also, the deviation from the asymptotic value of the spectral index  $n_s$  at small Higgs masses is smaller, than given by the one loop calculation. For low Higgs masses the spectral index drops, and goes out of the allowed region ( $n_s > 0.93$ ) for

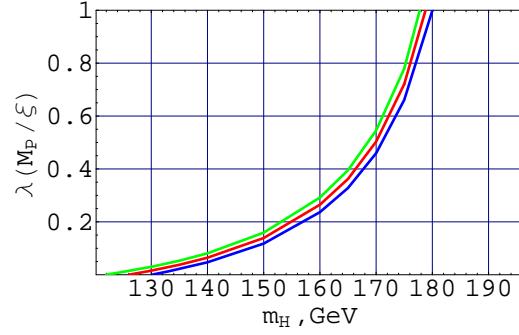
$$m_{\min}^{II} = \left[ 126.7 + \frac{m_t - 171.2}{2.1} \times 4.5 - \frac{\alpha_s - 0.1176}{0.002} \times 1.7 \right] \text{ GeV} . \quad (7.3)$$

This is a bit stronger, than (7.1).

It is interesting to note, that at large Higgs masses in any calculation the spectral index approaches the tree level value  $n_s = 0.967$ . The reason is that radiative corrections play a subdominant role if  $\lambda$  is not very small, and the running of the coefficient  $\lambda/\xi^2$  in front of the tree level potential is less important for large  $\lambda$  (see Eq. (5.11)).



**Figure 4:**  $\xi$  at the scale  $M_P/\xi$  depending on the Higgs mass  $m_H$  for  $m_t = 169.1, 171.2, 173.3$  GeV (from upper to lower graph). Solid lines correspond to prescription I, dashed — to prescription II. Changing the e-foldings number and error in the WMAP normalization measurement introduce changes invisible on the graph.



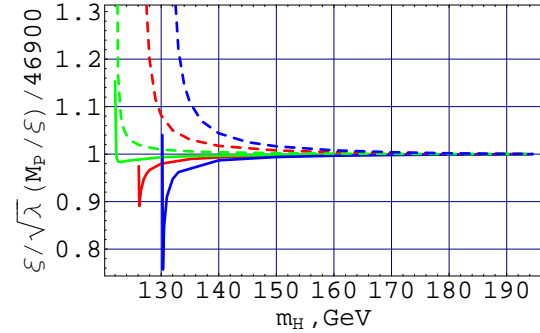
**Figure 5:**  $\lambda$  at the scale  $M_P/\xi$  depending on the Higgs mass  $m_H$  for  $m_t = 169.1, 171.2, 173.3$  GeV (from upper to lower graph). Changing the e-foldings number and error in the WMAP normalization measurement introduce changes invisible on the graph.

The comparison between different approximations for the effective potential can be found in Fig. 7. For this figure we always made two-loop SM running of the constants up to the  $M_P/\xi$  scale, to evade the overall mass shift and allow for comparison. Then, we plotted one-loop effective potential (4.1) and two-loop potential (4.2) *without* any further RG improvement; tree level potential with constants improved by chiral RG running up to the scale (6.1) (or (6.2) for prescription II), and RG enhanced tree level potential plus one- or two-loop potential.

In prescription I all results are almost indistinguishable, because the normalization scale (6.1) changes very little during the inflation.

In prescription II the results differ. We can see, that addition of the two-loop corrections changes the one-loop result rather strongly, because of accidental cancellation of the one-loop contributions from the gauge bosons and the top quark at one-loop. The RG enhanced result is rather close to the result obtained by using the two-loop effective potential. Furthermore, adding loop corrections to the RG enhanced tree-level result does not change the results much.

Our results are stable against variation of the parameter  $\kappa$ , defined in (6.1), (6.2), demonstrating that the running of the coupling constants is indeed compensated by the



**Figure 6:** The ratio  $\xi/\sqrt{\lambda}/46900$  at the scale  $M_P/\xi$ . The ratio equal to one [14] corresponds to the COBE normalization requirement for tree level potential. Solid lines are for prescription I, dashed — prescription II. Green, red and blue lines (from left to right) correspond to  $m_t = 169.1, 171.2, 173.3$  GeV.



logs in the effective potential (as it should be). In Fig. 8 we show the change of the different approximations to effective potential due to the change of  $\kappa$ . It is seen, that adding one- and two-loop logarithms to the RG-enhanced tree level potential removes the superficial  $\mu$  dependence of the potential.

The close similarity of the one-loop and two-loop results demonstrates that our conclusions are stable against radiative corrections for both renormalization procedures. The main effect of accounting of two-loop running and one-loop matching is the shift of the admissible interval for Higgs boson masses. However, the spectral index  $n_s$  cannot be determined unambiguously in the SM inflation by giving the exact values of the SM parameters (in particular the Higgs and top masses). Though  $n_s$  remains close to its tree value in the wide range of Higgs masses, (small) deviations from it contain an extra uncertainty, coming from the choice of the normalization point (cf. prescriptions I and II discussed in section 6). From theoretical point of view, this uncertainty can only be removed if the physics at Planck scales is fixed. A more exact cosmological measurement of  $n_s$ , together with precise knowledge of the SM parameters can help to resolve this issue.

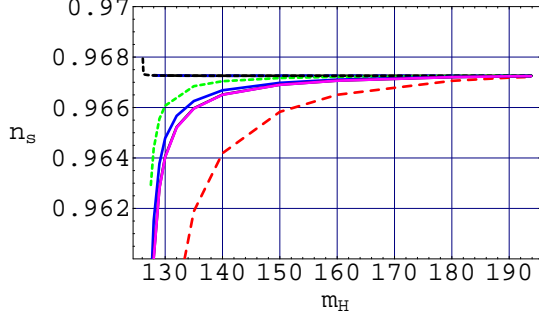
It is interesting to note that the requirement of successful Higgs-inflation allows to put an upper bound on the mass of the top quark. If t-quark mass were larger than 240-250 GeV, no choice of  $\xi$  and of the Higgs mass could lead to acceptable inflationary parameters. This number is smaller than the experimental lower constraint on the mass of the  $t'$ -quark of the fourth generation [46], meaning that Higgs-inflation in the SM is not consistent with fourth fermionic family.

We conclude this section with an estimate of the theoretical uncertainties related to higher order loop corrections<sup>12</sup>. To test the accuracy of the pole matching procedure at low energy scale one can use somewhat different procedures to compute the values of the coupling constants at  $\mu = m_t$ . First, we simply used the formulas given in [42, 43] for  $\mu = m_t$ . Then we used the same equations from [42] at  $\mu = m_Z$ , got the coupling constants at this point, and then considered their running up to  $\mu = m_t$ . The difference in the values gives an estimate of the uncertainties related to the two-loop terms in the matching procedure. For small Higgs masses in the region  $m_H \simeq 130$  GeV the variation in the scalar self-coupling is<sup>13</sup>  $\delta\lambda(m_t)/\lambda(m_t) \simeq 0.018$  and in the top Yukawa coupling  $\delta y_t(m_t)/y_t(m_t) \simeq 0.0015$  (uncertainties in other couplings are much smaller). This leads to the effective redefinition of the Higgs mass by 1.2 GeV and the top mass by 0.3 GeV. The change of the top mass by this amount leads to uncertainty in the lower limit on the Higgs mass equal to 0.6 GeV. The three-loop  $\alpha_s$  correction to the top mass gives  $\delta y_t(m_t)/y_t(m_t) \simeq 0.0042$  [47, 48], the non-perturbative QCD effects in the top pole mass -  $\overline{\text{MS}}$  mass matching are expected to be at the level of  $\delta y_t(m_t)/y_t(m_t) \simeq 0.001$  [49]. The four-loop  $\alpha_s$  contribution to the top mass was guessed to be the same [50]. If we assume that these uncertainties are not correlated, and symmetric, we get a theoretical error in the determination of the critical Higgs mass,  $\delta m_{\text{theor}} \simeq 2.2$  GeV. This error is smaller than the present uncertainties related to the experimental errors in the top mass and in the strong coupling constant. It can be reduced further to  $\delta m_{\text{theor}} \simeq 0.4$  GeV by the complete

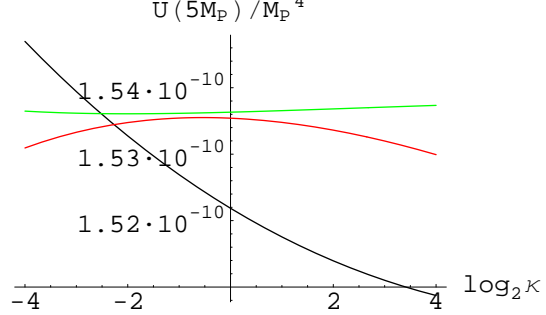
<sup>12</sup>We thank Mikhail Kalmykov for discussion of this issue.

<sup>13</sup>We give two significant digits for uncertainties in  $\lambda$  and  $y_t$  in order to allow for a detailed comparison.

two-loop pole matching (not known at the moment), accounting for 3-loop  $\alpha_s$  correction to  $y_t$ , found in [47, 48], and by the three-loop RG running up to the scale  $M_P/\xi$  (note that the complete 3-loop  $\beta$ -functions for the Standard Model are not known yet).



**Figure 7:** Spectral index  $n_s$  depending on the Higgs mass  $m_H$  calculated in different approximations. Nearly horizontal line represents the result in the prescription I (different approximations lead to indistinguishable results). Everything else is the results in prescription II. Red dashed line (lower) is the one loop effective potential, green dashed line (upper) is the two loop effective potential, close solid lines are RG enhanced results without (blue) and with addition of the one or two loop effective potential (red and black, nearly indistinguishable). For all graphs  $m_t = 171.2 \text{ GeV}$ ,  $\alpha_s = 0.1176$ .



**Figure 8:** Result of moving the normalization point (6.1) in the potential. The black falling graph is the tree level contribution, red (lower) and green (upper) demonstrate removal of the dependence on the normalization point. Potential is calculated in prescription I for the typical inflationary field  $\chi = 5M_P$  and  $m_H = 140 \text{ GeV}$ .

## 8. Comparison with refs. [1, 2]

Our preprint [1] appeared in hep-ph simultaneously with the paper [2], devoted to the study of the same problem. The analysis of [2] was carried out with the use of two-loop RG equations for the running couplings, modified to account for non-minimal coupling of the Higgs field to Ricci scalar in the Jordan frame. In [1] we used one-loop approximation and performed the computations of the effective potential in the Einstein frame.

In very general terms, the findings of [1], the present paper and of [2] are quite similar. Namely, in [1] it was shown that the Higgs-inflation in the SM, consistent with WMAP observations, is possible provided the Higgs mass  $m_H$  lies in the interval  $m_{\min} < m_H < m_{\max}$ , somewhat exceeding the region of the Higgs masses in which the SM is a viable effective field theory up to the Planck scale. In one-loop approximation the minimal and maximal Higgs masses were found to be

$$\begin{aligned} m_{\min} &= [136.7 + (m_t - 171.2) \times 1.95] \text{ GeV} , \\ m_{\max} &= [184.5 + (m_t - 171.2) \times 0.5] \text{ GeV} . \end{aligned} \quad (8.1)$$

The uncertainty resulting from variation of the strong coupling constant was not estimated in [1].

With the use of two-loop analysis, the paper [2] found that the successful SM inflation can take place if the Higgs mass exceeds the value

$$m_{\min} = [125.7 + \frac{m_t - 171}{2} \times 3.8 - \frac{\alpha_s - 0.1176}{0.0020} \times 1.4] \text{ GeV} \pm \delta, \quad (8.2)$$

where  $\delta \sim 2 \text{ GeV}$  indicates theoretical uncertainty from higher order corrections.

At the quantitative level, the one-loop value of [1] for  $m_{\min}$  is some 11 GeV larger than that of [2]. The result of our present paper for the minimal allowed Higgs mass is given in eqns. (7.1), (7.3) and almost coincides with the bound found in [2]. Therefore, the shift of the critical Higgs mass is associated with the account for one-loop corrections at the low-energy matching (instead of tree procedure) and with two-loop (instead of the one-loop) running from the low scale to the high scale. The existence of this shift due to higher order effects was anticipated in [1].

The behaviour of the spectral index as a function of the Higgs mass is a more subtle effect. The choice of the normalization point of [2] coincides with that of [27] and corresponds to the prescription II of [1] and of the present work. Let us compare our results for this case with those of [2].

According to [2], if  $m_H$  decreases,  $n_s$  increases and is always larger than the tree value (see Fig. 1 of [2]). According to computations of [1] and of the present work,  $n_s$  decreases and is always smaller than the tree value (Fig. 3 of [1] and our Fig. 1). The similar type of discrepancy exists for the behaviour of the tensor-to-scalar ratio (cf. Fig. 5 of [2] and our Fig. 2). The two-loop computation carried out in the present work shows that the difference between the results of [2] and [1] is *not* related to the number of loops accounted for. Below we will try to elucidate possible origins of this discrepancy. Making a detailed comparison of computations in both articles is difficult as we used different frames for analysis and solved different equations. However, there are several essential points which are treated differently in these works even in one-loop approximation.

- (I) The first point is related to gauge invariance. Our method (and the one used in [1]) is explicitly gauge-invariant. The  $\beta$ -functions for RG equations we used do not depend on the gauge choice, and the running of the couplings up to the point  $M_P/\xi$  is gauge independent. The boundary conditions are gauge-invariant as well. The method of [2] includes explicitly the gauge non-invariant object — the anomalous dimension of the scalar field  $\gamma$ . The equation (A.5) of this work indicates that the Landau gauge was used (in the arbitrary  $\alpha$ -gauge one would get

$$\gamma = \frac{1}{16\pi^2} \left( (2 + \alpha) \frac{3g^2 + g'^2}{4} - 3y_t^2 \right) + \dots \quad (8.3)$$

(here  $\alpha$  is the coefficient in front of the longitudinal term of the gauge-field propagator,  $\alpha = 1$  and  $\alpha = 0$  correspond to the Landau and Feynman gauges respectively). The gauge non-invariant parameter  $\gamma$  is used then for RG running of all coupling constant

through equations like  $d\lambda/dt = \beta_\lambda/(1 + \gamma)$ . Clearly, the definition of all couplings is then not gauge-invariant. It does not correspond to the standard gauge-invariant  $\overline{\text{MS}}$  prescription. Therefore, the pole- $\overline{\text{MS}}$  matching described in [43] is not appropriate for fixing the initial condition for the *gauge non-invariant* couplings used in [2]. The gauge variation of the scalar self-coupling  $\lambda$  (most important for computation of inflationary spectral indexes) reads

$$\delta\lambda = -\frac{\delta\alpha}{16\pi^2} \int^t dt \beta(\lambda) \left( \frac{3g^2 + g'^2}{4} \right), \quad (8.4)$$

where  $\delta\alpha$  is the change of the gauge parameter. It does introduce the gauge dependence of the results at two-loop level and may result in the change of  $n_s$  behaviour with the Higgs mass.

- (II) The second point is related to consistency of introducing of  $s$ -factor, which accounts for gravity-Higgs mixing in the Jordan frame. One can see that the argument leading to Eq. (4.6) of [2] is only applicable to the physical Higgs field and does not work for Nambu-Goldstone bosons present in the Landau gauge, used in [2]. Therefore, the kinetic term for these fields is not modified. In other words, the virtual Nambu-Goldstone bosons, contributing to diagrams for  $\beta$ -functions must be kept. This fact was accounted for in our paper (section 5) but not in [2], as the comparison of the one-loop parts of relations (A.1) and (A.2) and our equations in section 5 shows. One finds the differences in the running of the  $\text{SU}(2) \times \text{U}(1)$  gauge couplings  $g$  and  $g'$ , of the scalar self-coupling  $\lambda$  and of the top Yukawa coupling  $y_t$ . For example, the  $\beta$ -function for  $y_t$  does not contain any contribution from  $y_t$  in the inflationary region in [2], whereas it is in fact present. We suspect that this may be one of the reasons for the opposite behaviour of the spectral index  $n_s$  found in [2], which underestimates this contribution. Moreover, the running of extra couplings  $\alpha_0$  and  $\alpha_1$ , necessary for description of the electroweak theory in the inflationary region, was not considered in [2].
- (III) Yet another (numerically less important) difference is related to accuracy of computation. Reference [2] did not compute one- or two-loop effective potential but used the tree potential improved by the renormalization group. In this way the leading logs are accounted for, but the terms without logs are lost. In our present work all these terms up to the two-loop order are included.

The second version of [2] contains a “Note Added” in which a number of remarks about our work [1] has been made. We comment on them below. For the reader convenience, we quote here the most relevant part of the “Note Added” of ref. [2]:

“In our analysis, we computed the full RG improved effective potential. We did this including (i) 2-loop beta functions, (ii) the effect of curvature in the RG equations (through the function  $s$ ), (iii) wavefunction renormalization, and (iv) accurate specification of the initial conditions through proper pole matching. On the other hand, [1] did not compute the

full effective potential or include any of the items (i)–(iv).<sup>14</sup> Instead ref. [1] approximated the potential at leading log order with couplings evaluated at an inflationary scale after running them at 1-loop (this is one step beyond [27] where couplings were not run)."

We certainly agree with [2] that in [1] we did not include (i) 2-loop  $\beta$  functions (we had one-loop running) and (iv) one-loop pole matching for initial conditions for renormalization group (we used the tree values). We said explicitly that our aim is the *one-loop analysis* of the problem, for which the tree pole matching is adequate. As for the point (ii), the *one-loop* renormalization group running of the combination  $\lambda/\xi^2$ , relevant for inflation, is *exactly the same* at small and at large Higgs field values, meaning that our analysis is perfectly consistent at one loop. Concerning the point (iii), our formalism *does not* require the computation of the wave-function renormalization. At the same time, the running of  $\xi$  (which was not accounted for in [2]) plays an important role. See also the discussion of wave-function renormalization and pole-matching procedure in (I), and a comment on "full RG improved effective potential" in (III).

## 9. Conclusions

Inflation does not necessarily require the existence of the inflaton — the Higgs boson of the Standard Model can make the Universe flat, homogeneous and isotropic, and produce fluctuations necessary for structure formation. This happens in the SM with sufficiently large non-minimal coupling of the Higgs to the gravitational Ricci scalar. An important requirement is the validity of the SM all the way up to the Planck scale. In this case radiative corrections to the inflationary potential can be computed. They can be used to put constraints on the Higgs mass and to determine the connection between the cosmological parameters and properties of particles in the Standard Model. The window of allowed masses for the Higgs boson is  $m_H \in [126, 194]$  GeV (for central values of  $m_t$  and  $\alpha_s$  and neglecting theoretical uncertainties). It somewhat exceeds the region where the canonical SM is a valid field theory up to the Planck scale,  $m_H \in [126.3, 174]$  GeV. This prediction can be testable at the LHC. The precision measurements of  $n_s$  and  $r$ , together with exact knowledge of the SM parameters will allow to shed light on the origin of inflation and on Planck scale physics.

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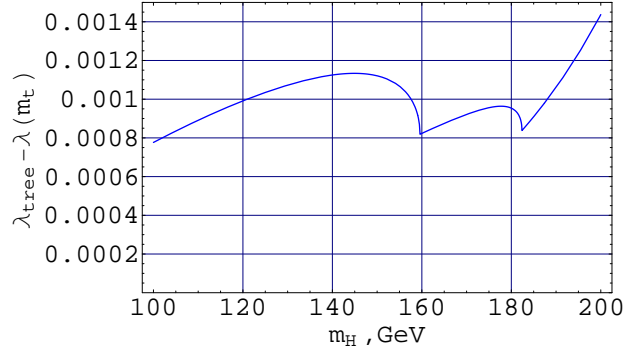
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## A. Initial conditions

The  $\overline{\text{MS}}$  values for the gauge coupling constants at  $\mu = m_t$  were obtained from the  $\mu = m_Z$

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<sup>14</sup>Though wavefunction renormalization was not included in [1], external leg corrections in the running of  $\lambda$  were included. However these two effects roughly cancel against one another.



**Figure 9:** Correction to the Higgs self coupling  $\delta_h = \lambda_{\text{tree}} - \lambda(m_t)$  depending on the Higgs mass  $m_H$ .

present in [46] by one-loop running for  $g'$ ,  $g$  and two-loop running for  $g_s$ . The resulting values for the gauge couplings at  $m_t$  are  $g' = 0.3587$ ,  $g = 0.6484$ ,  $g_s = 1.1619 + 4.5(\alpha_s - 0.1176)$ . Close values for electroweak constants were also obtained directly from the pole masses using formulas from [42].

The top quark Yukawa constant  $y_t$  and Higgs self-interaction  $\lambda$  at  $\mu = m_t$  can be obtained using the expressions from appendix of [43]. To summarize, the  $\overline{\text{MS}}$  value for the top Yukawa is very well approximated as

$$y_t(m_t) = \frac{\sqrt{2}m_t}{v}(1 + \delta_t), \quad (\text{A.1})$$

$$\delta_t = -0.0566 + \left( \frac{m_t - 171.2 \text{ GeV}}{6500 \text{ GeV}} \right) + \left( \frac{\alpha_S(m_Z) - 0.1176}{2} \right) - \left( \frac{m_H - 130 \text{ GeV}}{55000 \text{ GeV}} \right),$$

where  $v^2 \equiv \frac{1}{\sqrt{2}G_F} = (246.221 \text{ GeV})^2$ . The initial data for  $\lambda$  is  $\lambda(m_t) = m_H^2/2v^2 - \delta_h$ , where  $\delta_h$  is given in Fig. 9.

Using formulas from [42] similar values are obtained. The main difference in  $\delta_t$  comes from the two-loop QCD contributions accounted for in [43] and not in [42]. The difference in  $\delta_h$  comes from neglecting the W and Z mass difference in the loop integrals in [42], but accounted in [43]. Numerically, this just slightly changes the influence of the top mass on the overall results.

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